# Data Fusion of GPS Displacement and Acceleration Response Measurements for Large Scale Bridges

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July 11, 2006

# Outline

# Introduction



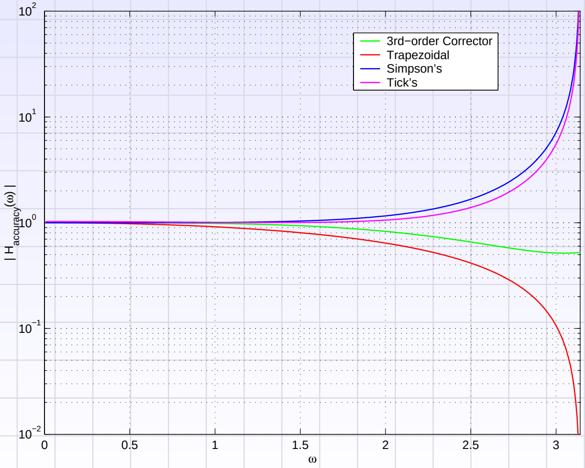
## Review of Integration Problem

$$\dot{x} = \int \ddot{x} dt + A$$

$$x = \int \dot{x} dt + At + B$$

- Typical method is to remove the mean $A$ from $\dot{x}(t)$, and again remove $B$ from $x(t)$
- Even after this, spurious low frequency error still exists in the displacement
- Could HP filtering data first, however, problems exist:
  1. Doesn't work for on-line applications
  2. Such filtering needs to be undertaken for zero-phase lag
  3. Removes significant low frequency components

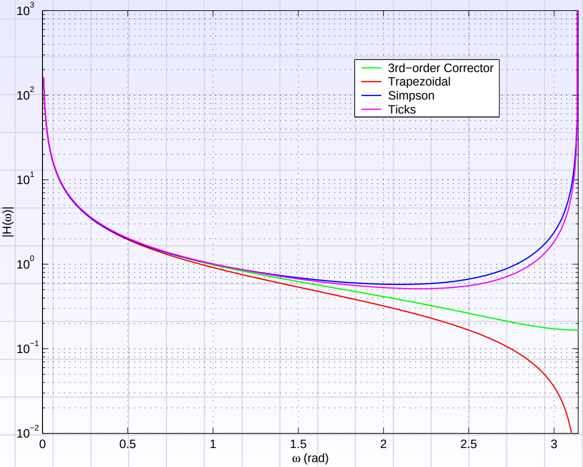
## Transfer Function Accuracy



# Time-Domain Integration Techniques



## Noise Amplification



## State-Space Modeling of the Measurement Processes

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ddot{x}_m + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \eta_a$$

$$z = x_m = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \eta_d$$

i.e.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{w}$$

$$z = \mathbf{H}\mathbf{x} + v$$

## Discrete Time State-Space Model

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T_a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} {T_a}^2/2 \\ T_a \end{bmatrix} u(k) + \begin{bmatrix} {T_a}^2/2 \\ T_a \end{bmatrix} \eta_a(k)$$

$$z(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \eta_d(k)$$

i.e.

$$\mathbf{x}(k+1) = \mathbf{A}_d \mathbf{x}(k) + \mathbf{B}_d u(k) + \mathbf{w}(k)$$

$$z(k) = \mathbf{H}\mathbf{x}(k) + v(k)$$

# Multi-rate Kalman Filter and Smoother

## Kalman Filtering

Time update:

$$\hat{\mathbf{x}}(k+1|k) = \mathbf{A}_d\hat{\mathbf{x}}(k|k) + \mathbf{B}_d u(k)$$

$$\mathbf{P}(k+1|k) = \mathbf{A}_d\mathbf{P}(k|k)\mathbf{A}_d^T + \mathbf{Q}_d$$

Measurement update:

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) + \mathbf{K}(k+1)\left[z(k+1) - \mathbf{H}\hat{\mathbf{x}}(k+1|k)\right]$$

$$\mathbf{P}(k+1|k+1) = \left[\mathbf{I} - \mathbf{K}(k+1)\mathbf{H}\right]\mathbf{P}(k+1|k)$$

Kalman gain $\mathbf{K}(k+1)$ is given by

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k)\mathbf{H}^T\left[\mathbf{H}\mathbf{P}(k+1|k)\mathbf{H}^T + \mathbf{R}_d\right]^{-1}$$

## Multi-rate Kalman Filtering

- Assume displacement and acceleration sampling intervals are $T_d$ and $T_a$ respectively, where $T_d/T_a = M$, $M$ is an integer
- Between the times $kT_d$, only the time update is performed and the optimal estimate is

$$\hat{\mathbf{x}}(k+1|k+1) = \hat{\mathbf{x}}(k+1|k) = \mathbf{A_d}\hat{\mathbf{x}}(k|k) + \mathbf{B_d}u(k)$$

$$\mathbf{P}(k+1|k+1) = \mathbf{P}(k+1|k) = \mathbf{A_d}\mathbf{P}(k|k)\mathbf{A_d}^T + \mathbf{Q_d}$$

- When displacement measurements are available at times $kT_d$, both the time update and measurement update are performed

## Smoothing

- Implement fixed-lag smoothing by using RTS fixed-interval smoothing algorithm
- First filtering up to the current measurement and then sweeping back a fixed number of steps $S$ with the RTS algorithm. If the number of backward steps is small, then the estimation is near "on-line"
- The smoothed estimates $\hat{\mathbf{X}}(k|N)$ over (0, N) is given by

$$\hat{\mathbf{X}}(k|N) = \hat{\mathbf{X}}(k|k) + \mathbf{F}(k)[\hat{\mathbf{X}}(k+1|N) - \hat{\mathbf{X}}(k+1|k)]$$

where smoothing gain $\mathbf{F}$(k) is given by

$$\mathbf{F}(k) = \mathbf{P}(k|k)\mathbf{A_d}^T\mathbf{P}^{-1}(k+1|k), \quad k = N-1, N-2, \ldots, 0$$

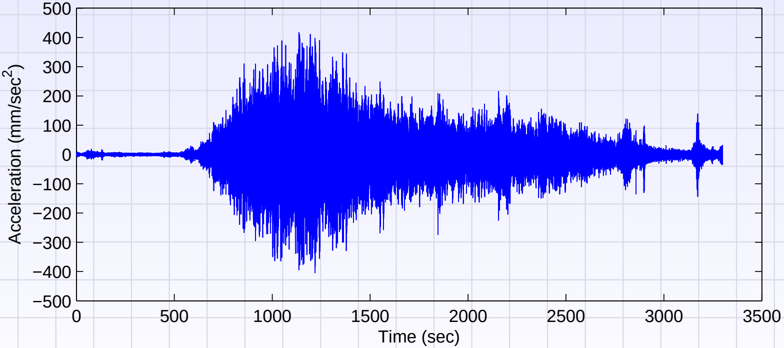
# Application to the Monitoring of a Large Scale Bridge

## Dynamic monitoring

- A large scale suspension bridge located near New York City has a main span of 1298 meters
- Displacement and acceleration were monitored during the 2004 NYC Marathon
- The sampling frequency of the acceleration measurement is 100 Hz
- The sampling frequency of the displacement measurement is 5 Hz, i.e., the Nyquist frequency is 2.5 Hz

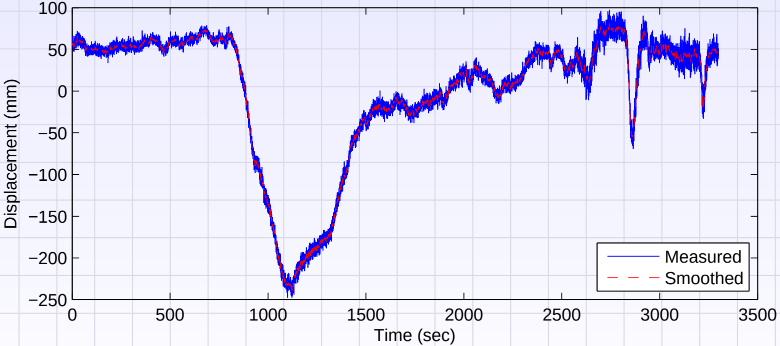
# Acceleration Measurements from the Bridge



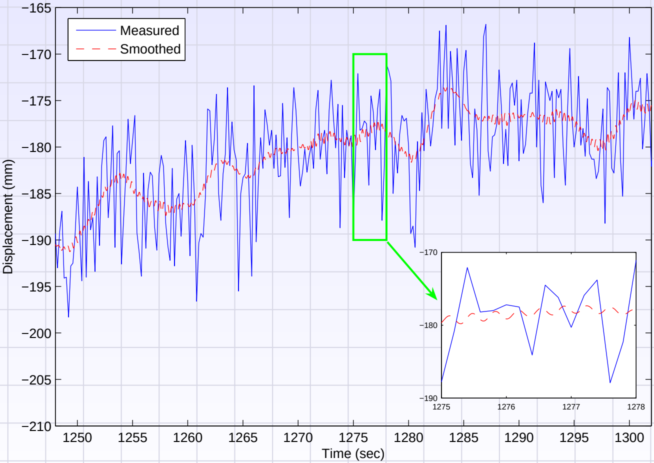


# Displacement Estimates



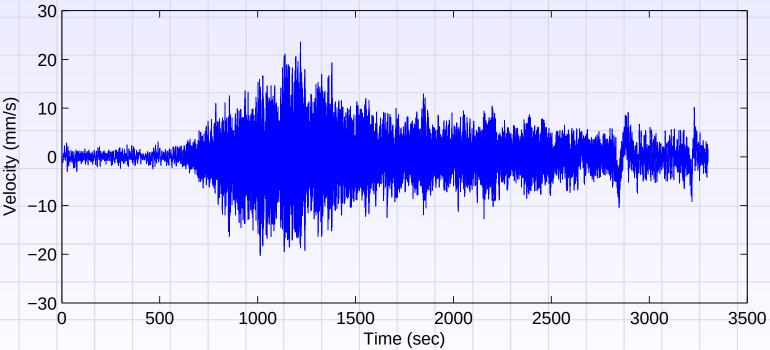


# Displacement Estimates



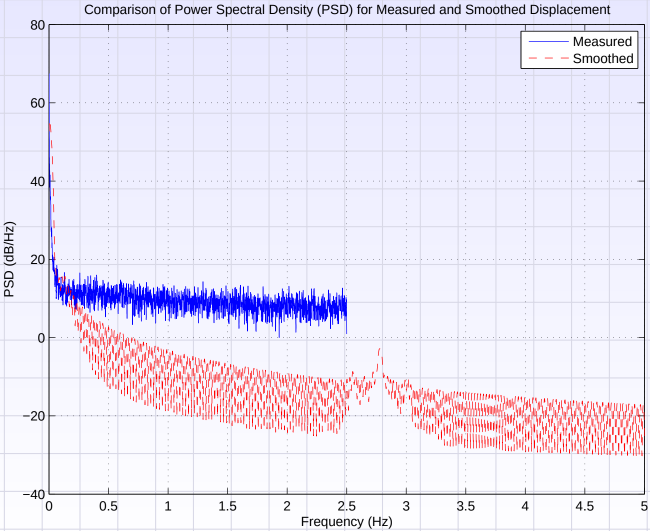
# Velocity Estimates





# Comparison of PSD for Measured and Smoothed Displacement





# Conclusions

## Conclusions

- The multi-rate Kalman filter and smoother provide a robust method to estimate displacement and velocity responses for large scale bridges.
- The multi-rate estimating aspect permits a relatively low sampling rate for GPS displacement measurements

# *Thank You!*